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AXIOMATIZATION OF DECLARATIONS
AND THE FORMAL TREATMENT OF AN
ESCAPE CONSTRUCT

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ONDERAFDELING DER TOEGEPASTE WISKUNDE
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ABSTRACT

We give Hoare-like axiomatizations for variables, procedures and a new escape construct. The proof rules treat scope rules properly; simplifications are shown for a restricted use of global variables and for a disallowance of innerblocks. It is shown that the axiomatization of the escape construct has influenced its design.

1. INTRODUCTION

In this section we give three criteria which should be fulfilled by any axiomatization and we extend the usual formulae expressing partial correctness by an environment component.

Some criteria

We feel that (some) existing axiomatic definitions are not fully satisfactory. This holds in particular with respect to the following criteria for the proof rules of the axiomatic system.

c1. The rules formalize the concept of "scope".
c2. The rules reflect and more importantly direct a (the) natural way of reasoning about the correctness of the program constructs.
c3. The rules do not necessitate any alteration of the program text and their applications can be written unambiguously within the program text.

Definitions of procedures by temporarily valid rules, viz relative to the scope of the procedure declaration as in Hoare & Wirth (1973), run the risk not to be a well understood formal system because temporal validity of proof rules is not well known in mathematical logic. The rules for procedures presented in Donahue (1976) are inconsistent because they do not treat the scope rules properly. Rules which model procedure call by body replacement conflict with c2 because procedures should be considered as abstractions rather than as abbreviations of a program text and consequently should be inspected just once. Rules which treat scope rules by renaming in the program text, Hoare (1971), Hoare & Wirth (1973), conflict with c3 (and c2). Donahue (1976) seems to be the first one explicitly requiring the first half of c3.

The environment component

The usual formalism of formulae \( \{p\}S\{q\} \) is not rich enough to give a satisfactory treatment of scope rules due to its inability to express the environment of a statement \( S \) and more importantly, to update the environment. Therefore the formulae of our axiomatic system have the format "\( \text{Env}\vdash\{p\}S\{q\} \)", to be read as "in the environment \( \text{Env} \) the behaviour \( \{p\}S\{q\} \) holds". \( \text{Env} \) is a set of formulae representing semantic properties of names which are statically true in \( S \); these properties may be described by the declarations in whose scope/range \( S \) is contained. The symbol \( \vdash \)
does not denote the deducibility relation but is merely used as a separation mark as in Gentzen-like calculi of sequents. The symbol $\vdash$ is used for deducibility in a Hoare-like system. The type of formulae which may be taken into $\text{Env}$ will be described when needed; some typical examples however are "var $V$ : int", indicating that $v$ is a variable of type int, and

$$\text{proc}(x = x_0 \land y = y_0) \text{exch}(\text{var } x, y \text{ : int } (\text{hyp } x_0, y_0 \text{ : int }))\{x = y_0 \land y = x_0\}$$

indicating that exch is a procedure which exchanges the value of its two parameters. In order to describe the updatings of $\text{Env}$ the following notation is used: $+$ and $-$ for set theoretic addition and deletion, $[\text{xxx}]$ for all formulae in $\text{Env}$ in which the main identifier is tagged with $\text{xxx}$ ($\text{xxx}$ is var, proc, esc, ...), $[\text{xxx id}]$ for all formulae of $\text{Env}$ in which the main identifier is id and is tagged with $\text{xxx}$, and $[\text{id}]$ for all formulae of $\text{Env}$ in which $\text{id}$ has a free occurrence. The notion of free occurrence is as usual: in particular in a formula expressing some semantics of a procedure, the formal parameter names, included those within the curly brackets and, bind the corresponding names in the entire formula. Bound names may always be renamed without affecting the meaning of the formulae. Finally, $[\text{id} + \text{id}']$ denotes the usual substitution operator which substitutes $\text{id}'$ for each free occurrence of $\text{id}$ - taking care of bound names $-$; simultaneous substitution is denoted by $[\text{id1}, \text{id2} + \text{id}' \text{, id2}']$.

The environment component clearly corresponds to the environment argument in denotational semantics, Tennent (1976), and to inherited attributes in attribute semantics, Knuth (1968). The keywords var, proc, ... correspond to injection as well as projection and inspection in a disjoint sum!

The concept of invironment has also been formalized in Clarke (1976) and Apt & de Bakker (1977). Both however treat the semantics of a procedure as a piece of program text, thus conflicting c2 and c3.

2. AXIOMATIZATION OF VARIABLES AND PROCEDURES

In this section we distinguish between various syntax properties. These strongly influence the formal rules for variables and procedures.

2.1. Unrestricted globality

The purpose of any declaration is to introduce a new entity, say with name $n$, and to state properties of it. So the environment must be updated with the static properties of $n$, nsem say, and potentially existing objects with the same name must be considered distinct.

$$\ldots \text{ possibly another premise justifying nsem } \ldots$$

$$D_1 \text{ Env} \leftarrow \{\text{R} \text{new decl-of-n in stmtt ni(S)}\}$$

where $n'$ does not occur free in Env and n (and consequently R and S).

Some of the static properties of $n$, nsem, might be clear from the text (the type and the fact that it is a var or proc), others might require an additional proof (the net effect of a proc) and give rise to the other premise.

Variable declaration.

Thus the rule for variable declaration reads

$$V_1 \text{ Env} \leftarrow \{\text{R} \text{new var v:tv in stmtt ni(S)}\}$$

where v' not free in Env, v.

This should be compared with the usual axiomatizations as in Hoare & Wirth (1973):

$$V_2 \{\text{R} \text{new var v:tv in stmtt ni(S)}\}$$

where v' not free in R, S and, except for v$\in v'$, not free in stmtt either.
According to the latter rule the innerblock program writer is forced to choose new names in the course of the correctness proof whereas - hopefully - he had chosen the best names you can think of in the original program text. In the former rule however a new (read meaningless) name has to be chosen for an entity which is in the inner block much more meaningless than the variable v. Moreover, according to the latter rule the correctness proof cannot easily be written within the original program text, whereas according to the former rule we can place assertions in the program text in a natural way such that they unambiguously determine the relevant data for the formal rule:
\[ \vdash \text{[R]} \text{ new var } v^{(v')} : t v \text{ in } [R[v + v']] \text{ stmtt } \{S[v + v']\} \text{ ni } \{S\} \]

with a suitable convention of how surrounding declarations determine the environment.

**Initializing variable declaration**

Because the declaration has "side effects" the initial state for stmtt differs from \([R[v + v']]\) and we also need a second premise:

\[
\begin{align*}
\text{Env} & \vdash R \Rightarrow R'[v, v' + \text{exp}, v] \\
\text{Env}[v + v'] & + \text{var } v : t v \vdash [R] \text{ stmtt } \{S[v + v']\}
\end{align*}
\]

\[\text{v} \vdash (R) \text{ new var } v : t v = \text{exp} \text{ in stmtt } \text{ni } \{S\} \]

where \(v'\) not free in Env, v.\(^*\)

Note that it is very clearly stated that the declaration is nonrecursive: exp is interpreted in Env in stead of Env \([v + v']\).

**List of (Initializing variable) declarations**

A number of objects may be declared at a time in two ways: either simultaneously, separated by commas, or sequentially, separated by semicolons. It is very important that the defined names are distinct and so the construct is not merely a nest of blocks each one having a single declaration. Indeed, in the following rules the substitutions are ill defined if \(v, v'\) do not constitute a list of distinct names and moreover the restoration of the original values of \(v\) (performed by the transition from \(S[v + v']\) into \(S\)) is done at once for all members of \(v\) together. In the simultaneous case we have

\[
\begin{align*}
\text{Env} & \vdash R \Rightarrow R'[v, v' + \text{exp}, v] \\
\text{Env}[v + v'] & + \text{var } v_i : t v_i \text{ (i = 1 ... n)} \vdash [R_i] \text{ stmtt } \{S[v + v']\}
\end{align*}
\]

where \(v'\) is a list of distinct names not occurring free in Env, v and the sequential case (denoting \(v_1 \ldots v_i\) by \(v_{1...i}\)):

\[
\begin{align*}
\text{Env}[v_{1...i-1} + v_{i-1} v_{1...i-1}] & + \text{var } v_j : t v_j \text{ (j = 1...i-1)} \Rightarrow R_{i-1} \Rightarrow R_i[v_{1...i-1} v_{1...i-1}, (i=1...n)] \\
\text{Env}[v + v'] & + \text{var } v_j : t v_j \text{ (j = 1...i)} \vdash [R_i] \text{ stmtt } \{S[v + v']\}
\end{align*}
\]

\[\text{Env} \vdash \{R_0\} \text{ new var } v_{1...i} : t v_{1...i} = \text{exp}_{1...i} \text{ in stmtt } \text{in } \{S\} \]

where \(v'\) distinct and not free in Env, v.

Written within the program text the latter looks like

\[\{R_0\} \text{ new var } v_{1...i} : t v_{1...i} = \text{exp}_{1...i} \text{ in } [R_n] \text{ stmtt } \{S[v + v']\} \text{ ni } \{S\} \]

**2.2. Restricted globality**

We restrict globality in the following way. First define (recursively): a name

\* A deduction of the first premise requires many more rules which will not be given in this paper. The intuitive interpretation however is that \(R \Rightarrow R'[v, v' + \text{exp}, v]\) should be proved using facts from Env.
n is used globally in a text T if either n occurs free in T or n is used globally in the declaration of a procedure occurring free in T. Recall further that the range of a declaration is, informally, the text for which the declaration is valid and that the scope of a declaration is the range with exclusion of all those subranges which belong to a declaration of an object with the same name. Then define: globalization is restricted if no name is used outside its scope. The restriction disallows the following typical example:

new var v; proc p = (v:=0) in ... new var v in p ni ... ni.

In general, all those programs are disallowed whose semantics will be affected by extending the formal and actual parameter list by the global variable names, or equivalently, by adding the <nomenclature> of Dijkstra (1976) to each block. With restricted globality new objects really hide existing objects with the same name and so the updating of Env should have the form Env-[n] in stead of Env[n+n']. Although an outer var v is "nonexistent" in an innerblock new decl-of-v in ... ni its relation to other variables may be changed in the innerblock as shown by \{v<\w\} new var v in \w:=\w-1 ni \{v<\w\}. This necessitates the hypothetical constants in the scheme for declarations with restricted globality:

--- possibly another justifying nsem ---

\[
D_2 \quad \text{Env} \vdash (\exists n':tn'. RAI) \quad \text{new decl-of-n in stmtnt ni} \quad \exists n':tn'. SAI
\]

where

a. n' is a possibly empty list of distinct names,
b. no variable used globally in stmtnt is free in I,
c. n is not free in R, S and n'.

(By convention the range of \exists and later on \forall extends as far right as possible).

Thus I comprises all relations of interest whose variables can not be subject to change in stmtnt (and in particular fixes the relations of n' to these variables). In other words: the transition from S via ni to \exists n':tn'.SAI restores the original values (better: relations) of the variables which have been made invisible within stmtnt.

Although the rule might look more complicated, it actually prescribes a simpler way of reasoning, because it formalizes both the invisibility of an outer n from within the block and the invariance of some relations I. Thus criterion c2 has been satisfied. When the list n' is empty, \exists n':tn'.A is understood to be A, for any assertion A.

( Remark. The following adaption of scheme D1,

--- possibly another premise justifying nsem ---

\[
D_2 \quad \text{Env} \vdash (R[n+n']) \quad \text{stmtnt \{S[n+n']\}}
\]

where n' not free in Env,n

fails, because nothing is known about n' from within stmtnt, whereas its type is needed in proving an implication involving n').

Here is an example of the use of (D2) for variables:

let the current environment contain var v,w:int,
\{0<v<v\} \exists v':nat, (v'<v) \land (0<v'v'\w=v')\}

new (v':nat,) var v:int in \{v'<\w\} v:=w;v:=v-1; w:=v \{v'<\w\} ni
\{v'<nat, (v'<\w) \land (0<v'\w=v')\}\{0<v<\w\}.

---

* The rule requires the pre- and postconditions to be of a particular format and even prescribes the choice for the bound names n'. This strange feature can be eliminated by replacing the pre- and postconditions by Ro and So and adding the premises Env \vdash Ro \Rightarrow (\exists n':tn'.RAI) and Env \vdash (\exists n':tn'.SAI) \Rightarrow So.
Procedures with restricted globality

Consider the following syntax for recursive procedure declarations:

\[
\text{new proc } p \ (\text{spec } f:tf)=\text{body in stmt ni}, \ \text{where spec } f:ft \ \text{stand for a list of specifications, viz. var } x:tx \ \text{and val } y:ty, \ \text{indicating the Pascal variable and value parameter mechanism. Assume further that globality is restricted, a natural way of specifying some semantics of a procedure } p, \ \text{psem say, is the format}\]

\[
\text{proc } \{\!\{P\!\}\!\} \ p \ (\text{spec } f:ft\{\text{hyp } h:th\})\{\!\{Q\!\}\!\}\]

\text{where } P \ \text{and } Q \ \text{describe a net effect of } p \ \text{in terms of the formal parameters } f, \ \text{the hypothetical constants } h \ \text{and the global variables of } p, \ \text{i.e. the variables globally used in the declaration of } p. \ \text{Note that the global variables are precisely the free variable names in psem. The formal val parameters are considered as local variables and therefore may not occur free in } Q \ \text{(restr.1). The global variables are considered as implicit var-parameters and must have a free occurrence in the entire formula in order to have a consistent and concise interface between the rules for declaration and call (restr.2). The hypothetical constants may be used to relate the initial and final values of the (implicit and explicit) var-parameters, e.g.}\]

\[
\text{proc } \{x=x_0-1\} \ \text{incrx2}\{\text{hyp } x:0; \text{int}\}\{x=x_0+1\} \ \text{and must of course be distinct from } f \ \text{(restr. 3). In the sequel, these three restrictions are tacitly assumed. We assume in addition the following syntax property: in each call all var-parameters (i.e. the explicit actual var parameters and the variable names free in psem) are distinct.}\]

So let psem abbreviate \(\text{proc } \{\!\{P\!\}\!\} \ p \ (\text{spec } f:tf \{\text{hyp } h:th\})\{\!\{Q\!\}\!\}\).

The rule for procedure declaration then reads:

\[
\begin{align*}
\text{Env} - [\!\{P\!\}] + \text{psem} - [f,h] + \text{var } f:tf + h:th \vdash \{\!\{P\!\}\!\} \ \text{body in stmt } ni \quad (3p:tp'.S\text{AI})
\end{align*}
\]

where the restrictions on \(p', R, S, I\) are obvious from D2.

And the rule for procedure call is an axiom rather than a rule; it closely resembles the rule for blocks:

\[
\begin{align*}
\text{PCI: Env + psem } \vdash \{3h:th, P[f+a]\land I\} \ p(a)\{3h:th, Q[f+a]\land I\}
\end{align*}
\]

where all var-parameters not free in I.

Again, I comprises all relations of interest whose variables are not subject to change within procedure p. The way one actually reasons (should reason?) about procedures might be slightly better reflected in these rules than e.g. those in Hoare (1971) and Hoare & Wirth (1973). The rules have moreover the advantage that their application can unambiguously be written in the program text, see appendix A.

The following examples show that it is really necessary to have a means of deleting information from the environment - if no alteration of the program text is allowed -:

\[
\begin{align*}
\text{new proc zero } = (y:=0) \ \text{in new proc zero } = (y:=1) \ \text{in zero ni ni}, \\
\text{new proc zero } = (\text{new proc zero } = (y:=1) \ \text{in zero ni ni}, \\
\text{new proc zero } = (y:=0); \ \text{proc } p = (\text{new proc zero } = (y:=1) \ \text{in zero ni ni}) \ \text{in } p \ \text{ni.}
\end{align*}
\]

For each of these blocks the behaviour \{true\} \rightarrow \{y=0\} might wrongly be provable, whereas \{true\} \rightarrow \{y=1\} is right. Thus the rules presented in Donahue are inconsistent, the failure in the soundness proof being a wrong translation from the axiomatic formula into the denotational one.

Remark

One might feel it more natural to use the formal val-parameters in the post-condition of psem while expressing their initial values. Consider for example (a) \(\text{proc } \{y=0\} \ \text{assign } (\text{var } x \ \text{val } y)\{\text{hyp } y_0\}\{x=y_0\}\) with (b) \(\text{proc } \{\text{true}\} \ \text{assign } (\text{var } x \ \text{val } y)\{x=y\}\). But because we assume the PASCAL val-parameter mechanism, a call like \(\text{assign}(v, v+1)\) is allowed. Therefore we feel (b) slightly misleading. It is however quite well possible to allow (b), but then the rules have to force the introduction of additional hypothetical constants, \(y_0\) say, in the justification.
of not only psem but also the correctness of a call of p. It will appear that
the correctness proofs as directed by these rules will be quite the same.

Disallowing innerblocks

The language PASCAL disallows innerblocks. All local entities must be declared
at the entry of a procedure body (hence the keyword new is superfluous). This
may look rather inconvenient and one may wonder whether any semantic simpli-
cation has been gained. It turns out that the rule for declarations may be sim-
plicated if in addition the following, quite reasonable, syntax property is assumed:
for each procedure the {distinct} locally declared names are distinct
from the formal parameter names and also distinct from the global variable names
(which due to the restriction on globality is certainly satisfied when there are
no initializing variable declarations). They need not necessarily be distinct
from the formal parameter names and local names of the local procedures. Here
is the scheme for the justification of a procedures semantics:

--- possibly another premise justifying n_i sem --- , (i=1..m)

\[ \text{Env}[n] + n_i \text{ sem } (i=1..m) \vdash \{R'\} \text{ stmtt } \{S\} \]

\[ \text{Env} \vdash \{R\} \text{ decl-list-of-} n \text{ in stmtt } n_i \{S\} \]

(where n not free in R,S).

(R' equals R except when there are initializing variable declarations.) Thanks
to the restriction “n not free in R and S” the rule is also sound when innerblocks
are allowed or the distinctness property does not hold; but then the rule is no
longer complete, as shown by the impossibility to prove by (D3)

\[ \{v=0\} \text{ new decl-of-v in } \vdash n_i \{v=0\} \text{ and } \]

\[ \text{proc } \{x=0\} p(\text{var } x: \text{int})\{x=0\} = (\text{var } x: \text{int in skip } n_i) \]

Indeed, actually each name in the precondition R of the procedure body is
necessarily a formal or global parameter, hence distinct from n, and therefore
cannot be made invisible and consequently needs not to be “restored” upon
passing the closing n_i.

Here we specify (D3) for the case that the list of declarations is a sequential
one of initializing variable declarations followed by procedure declarations

\[ \text{Env}[v_p] + \text{ var } v_j : t_j \{j=1..i-1\} \vdash R_{i-1} \rightarrow R_i [v_i + \text{exp}_i ] \]

\[ \text{Env}[v_p] + \text{ var } v_j : t_j \{j=1..m\} + p_j \text{ sem } (j=1..i) \]

\[ -f_i , h_i \vdash \text{ var } f_i : t_f \vdash h_i : \text{th}_i \vdash \{p_i \} \text{body}_i \{Q_i\}, \{i=1..n\} \]

\[ \text{Env}[v_p] + \text{ var } v_j : t_j \{j=1..m\} + p_j \text{ sem } (j=1..n) \vdash \{R\} \text{ stmtt } \{S\} \]

\[ P_2 \text{ Env} \vdash \{R\} [\text{var } v_0 ; \text{var } v_i : t_i = \text{exp}_i ; \ldots ; \text{proc } : p_i (\text{spec } f_i : t_f) = \text{body}_i \ldots \text{in stmtt } n_i \{S\} \]

(where x,p not free in R_0,S).

Thus the proof of the procedure body breaks down to (m+1)n+1 independent subproofs.

2.3. Procedures with unrestricted use of global variables

The main problems caused by the use of global parameters - in their full glory -
is that distinct objects may be known under the same name: how then to describe
the semantics of a procedure? Consider for instance the declaration

\[ \text{proc p (var x:int)=zero in an environment which contains var x:int and } \]

\[ \text{proc } \{true\} \text{ zero } (x=0) \]

It seems that a change in the syntax of the language is necessitated in order
to obtain a definition satisfying the criteria. Thus assume that a declaration of
procedures looks like \( \text{proc } p(\text{spec } t_f) = \text{spec } f_i : t_f \text{ in body corp } (\text{cfr Algol 68}) \)
where only the type of the procedure is specified in its heading and not the
formal parameter names. Then we are free to choose new names, \( f' \) say, as fancy
formal parameters in the semantics psem so that the globals, \( f \), can have their
own name. It is only in the range of spec \( f : t_f \) that the globals \( f \) should be
renamed, into f" say, and the fancy formal parameter names f' should be renamed into f:

Let psem abbreviate \( \operatorname{proc}[P]p(\operatorname{spec}(f') : tf(\text{hyp } h : \text{th}))\{q\} \)

\[
\text{Env}[p+p'] + \text{psem}[f+f'\rarr] + \var f : \text{tf} + h : \text{th} |- \{P[f,f'+f",f]\} \text{ body } \{q[f,f'+f",f]\} \\
\]

\[
P3 \quad \text{Env} |- (R) \text{ new proc } p \ (\operatorname{spec}: \text{tf}) \= \operatorname{spec} f : \text{tf in body \operatorname{corp} in stmnt ni } \{S\} \\
\text{where } p' \text{ not free in Env, } p \text{ and } f", h \text{ distinct and not free in Env, } P, p', f. \\
\]

Again, it is easy to write it unambiguously within the program text:

\[
\ldots \ (R) \text{ new proc } p \ (\{p\} : \{p\} (\operatorname{spec}(f') : tf (\text{hyp } h : \text{th})) \{q\} \\
\quad \quad = \operatorname{spec} f \ {\rarr} \{f\} : \text{tf in } \{P[f,f'+f",f]\} \text{ body } \{q[f,f'+f",f]\} \text{ corp} \\
\text{in } \{R[p+p']\} \text{ stmnt } \{S[p+p']\} \text{ ni } \{S\} \ldots \\
\]

Of course, \( \{+p'\} \) and \( \{+f"\} \) could be omitted if p and f do not exist in the environment of the block.

2.4. Conclusion

It has been shown that scope rules may be nicely formalized in an axiomatic definition satisfying the criteria c1, c2, c3. This one has a measure to judge the mental complexity of the various program constructs (in this section: "syntax restrictions"). In particular it has appeared that unrestricted globality gives rise to rather complicated substitutions and also necessitates a change in the syntax, whereas restricted globality yields no problems at all. Moreover it seems that no further simplification may be gained by further restricting globality! This sheds another light on "Global Variables Considered Harmful", Wulf & Shaw (1973). A sequential list of declarations has appeared to be a construct that should be distinguished from a nest of blocks. The disallowance of inner-blocks together with some reasonable syntax assumptions really simplify the semantics.

The more specific deletions from Env, viz \([-\var v\text{ and } -\operatorname{proc} p\) should be used when (and allow that) in the same context a name may denote both a \( \var \) and a \( \operatorname{proc} \). This feature for instance should be used to formalize functions where the \( \text{fun} \) identifier occurring in the body also denotes a \( \var \). The next section shows another application.

In general we would like to take all static properties into the environment in stead of the pre- and postconditions. This might cause some troubles when the ranges are not properly nested, as for instance the <active scope> and <passive scope> of variables in Dijkstra (1976).

3. THE FORMAL TREATMENT OF AN ESCAPE CONSTRUCT

In Bron et al (1976) we have defined the so called escape mechanism for dealing with abnormal termination and abortion of program parts in a highly structured way. The escapes can be used to react to user defined events as well as to system generated error occurrences. They are efficiently implementable and in harmony with the block concept. A fuller treatment - except for the formal semantics - can be found in Bron et al (1976).

In this section we give the axiomatic definitions of the construct as originally designed, but with a slightly modified syntax. It will appear that at least criterion c3 has not been satisfied. An attempt to force the satisfaction of c3 will lead to an improvement.

3.1. The original escape construct

Here is a (rather operational) definition of the escapes.

(a) The block \( \text{new esc } e(\text{val:ty) in stmnt ni} \) declares an escape variable e.
As usual the declaration binds the name e in stmtt (and makes the programmer aware of the fact that abortions may take place within stmtt). In the scope of the declarations there may occur calls and several definitions of escape procedures e.

(b) The block def escproc e = val y in body corp in stmtt ni defines an escape procedure e. The type of the formal value parameters y is fixed by the declaration of escape variable e. The semantics of an escape procedure differs from normal procedures in that termination of the body causes the block to which the definition is local to be terminated (thus causing abortion of stmtt and abnormal termination of the block).

(c) Escape procedure calls read the same as procedure calls. Semantically they differ from the usual procedure calls in that their execution causes the "dynamically active" escape procedure to be executed. In other words, escape procedure definitions have dynamic in stead of static scope. (Hence var parameters have been disallowed).

(d) By default the escape variable declaration also means the definition of escproc e = val y in abort corp. Abort is a standard escape variable for which it is assumed that a suitable escape procedure has been defined surrounding the user program.

This completes the definition.

Thus the (implicit) insertion of an escape procedure call can not invalidate the textually succeeding assertion (because assertions have to be interpreted as being true whenever control reaches them). If however the programmer wants to continue an aborted computation anyway, then he should provide some surrounding block, having a weak enough postcondition, with an escape procedure definition. Appendix c gives a realistic example. If no escape procedure definition has been provided by the programmer, abort is called; it will abort program execution and may give an error message and a dump of the stack and the like.

3.2. The axiomatization

Apart from the axiomatization of our escape mechanism, this section is also interesting because of the treatment of "jumps out of procedures". In some axiomatizations of goto-like constructs, such jumps have been disallowed, Donahue (1976), whereas in others true enough they have not been disallowed but neither have procedures been axiomatized (in a way satisfying c2), Kowalski (1977).

In still others there arise scope problems which have not explicitly been dealt with, Clint & Hoare (1972).

As an important, first step we define the formats and some abbreviations expressing the semantics of (a) escape variables, (b) procedures and (c) escape procedures.

(a) The semantics of an escape variable is expressed by esc e(val:ty).

(b) The semantics, psem say, of a procedure is expressed by

\[ \text{psem} \equiv \text{proc}(p \circ \text{spec}(f); tf \ [\text{hyp} h: \text{th} \ e_1, \text{assem}(i=1\ldots n)](q), \text{where} \]

\[ e_1, \text{assem} \equiv \text{escproc}(E_1)e(y_1)). \]

The assumed semantics \( e_1, \text{assem} \) indicate - in terms of the bound names \( y_1 \), the free names \( f, h \) and globals of psem - the precondition \( E_1 \) on which the escape procedures \( e_1 \) will be called in the body of p and subsequently control will leave the body. Note that \( y_1 \) is bound in \( e_1, \text{assem} \) and that the binding of \( f \) and \( h \) extends over \( e_1, \text{assem} \). The three restrictions of section 2.2 are still tacitly assumed.

(c) The semantics, esem say, of an escape procedure is expressed by

\[ \text{esem} \equiv \text{escproc}(E)e(y' e_1, \text{assem}(i=1\ldots n)), \]

where \( e_1, \text{assem} \) is as above. Rather that expressing a condition transformation, like psem does, esem expresses a particular condition, viz. the (weakest) precondition upon which its body establishes the block's postcondition. Hence global variables of the same name as the formal parameters, \( y \) say, of its definition might be involved, even if they are not used globally in its body! (Thanks to a suitable choice of syntax esem can be written within the program text in a natural way.) E is called the precondition of esem.
The semantics of the escape procedure defined by default has been derived from the axiomatization of abort: true is a sufficient precondition because the programs postcondition will not be reached anymore. So
\[ \text{inutesem} = \text{escproc} \{ \text{true} \} \{ ( ) \} . \]

In the rules below we do not explain what should be clear from section 2. Because the free variable names of esem are not merely the names of the variables globally used in its body, we will not assume that globality has been restricted.

The rule for escape variable declaration is not surprising, cfr D1,
\[ \text{Env}[e+e'] + \text{esc e'(vality)} + \text{inutesem} \vdash \{ \text{R}[e+e'] \} \text{stmtt} \{ \text{S}[e+e'] \} \]
\[ \text{Esc} \]
\[ \text{Env} \vdash \{ \text{R} \} \text{new esc e(vality) in stmtt ni} \{ \text{S} \} \]
where e' not free in Env, e.

The rule for procedure declaration, cfr D1 and P4,
\[ \text{Env}[p+p'] - \{ \text{escproc} \text{psem} \} \{ f+f' \} \text{var f:tf + h:th + e,assem(i=1..n)} \]
\[ \text{stmtt} \{ \text{Q}[f,f+f''+f'] \} \]
\[ \text{P4} \]
\[ \text{Env} \vdash \{ \text{R} \} \text{new proc p (spec:tf) = spec f:tf in body corp in stmtt ni} \{ \text{S} \} \]
where p' not free in Env,p and h, f'' distinct and not free in Env,p,p',f.

Thus all escapes globally used in body give rise to explicit assumptions e,assem in psem: each escproc in Env has been deleted in the environment for body!1

The rule for escape procedure definition
\[ \text{Env} - \{ \text{escproc} \} \{ y+y'' \} \text{var y:ty + e,assem(i=1..n)} \vdash \{ \text{E}[y,y'\text{+}y'',y]\} \text{body(S[y,y'])} \]
\[ \text{E1} \]
\[ \text{Env} - \{ \text{R} \} \text{def escproc e = val y in body corp in stmtt ni} \{ \text{S} \} \]
where y' not free in Env,y.

Thus the justification of esem essentially requires to show that its body establishes the block's postcondition: after termination of body control will goto the end of the block. Note the resemblance with and deviations from D1 and P4. (The following justification of esem does not affect the semantics: replace (Env-[escproc]) by (Env-[escproc] + esem)).

The rules for escape procedure and procedure call. Assume without loss of generality that the bound names of e, assem will not occur free in I,a.
\[ \text{Env} + \text{esem} \vdash \text{escproc} \{ \text{E}[f+a] \text{AI}\} e(a) \{ \text{false} \} \]
\[ \text{EC1} \]
\[ \text{Env} + \text{esem} \vdash \{ \text{E}[f+a] \text{AI}\} e(a) \{ \text{false} \} \]
where no actual (global!) var-parameter occurs free in I and
\[ \text{Env} + \text{psem} \vdash \text{escproc} \{ \text{h:th.E}_i[f+a] \text{AI}\} e(a) \{ (y_i) \} \]
\[ \text{PC1} \]
\[ \text{Env} + \text{psem} \vdash \{ \text{h:th.E}_i[f+a] \text{AI}\} e(a) \{ (y_i) \} \]
where no actual var-parameter occurs free in I.

So the only difference with rule PC1 is the addition of the premise: it requires to verify in the current environment the actual assumptions, formally made in the justification of esem and psem. The assumptions have been weakened by the addition of the invariant I to their preconditions.

Finally we give two rules, called Semantics verification, enabling us to deduce the premise of EC1 and PC2. The first one is a trivial axiom:
\[ \text{SI: Env} \vdash e_i \text{sem}(i=1..n), \text{provided } e_i \text{assem}(i=1..n) \in \text{Env}. \]

The second one is also easy to formulate:
let Env contain esc e'_i (val:ty_i) and also
escproc {E_i}e_i{(y_i \ e_i{(y_i)})(j=1..n_i)}(i=1..n)
and assume without loss of generality that y_i will not occur free in I_i,
then,
Env \vdash \forall y_i:ty_i.E_i \rightarrow E_i , (i=1..n)
Env \vdash \forall y_{i_j}:ty_{i_j}.E_{i_j} \rightarrow I_{i_j} , (i=1..n)
Env + escproc {E_i}e_i{(y_i)}(i=1..n) \vdash escproc {E_{i_j} \wedge I_{i_j}}e_{i_j}{(y_{i_j})}(j=1..n_i,i=1..n)
Env \vdash escproc {E_i}e_i{(y_i)}(i=1..n)
where no variable globally used by e_i occurs free in I_i.

Thus in order to verify some "weakened e:assem" one has to prove that its precondition E_i implies the one of e:assem already present in Env and one has to verify suitably weakened versions of the assumptions on e_{i_j} (j=1..n_i) made by e_i. Because of recursive activation the environment component in the third premise has been extended by what is to be verified. Note that the third premise either invokes S1 or invokes - recursively - S2 again; the number of invocations however is bounded by the cardinality of the set of mutually dependent escape procedure semantics in Env. Appendix B contains a sample proof fully exploiting the recursive character of S2.

Any attempt to associate a unique condition with each escape variable will result in incompleteness, as shown by the following example:
new esc e() in def escproc e = ... {S1} corp in v:0;{v=v0} e () ni {S1} ...
def escproc e = ... {S2} corp in v:0;{v=v0} e () ni {S2} ...
This should be compared with usual variables; also with them it is impossible to associate a unique value.

3.3. Simplifying the semantics

Although the definition in terms of normal procedures, 3.1, looks quite simple, it might be argued that the escape mechanism is too complicated: the rules of semantics verification, S1 and S2, are rather exceptional. Not only because their right hand side is the semantics of a name in stead of the behaviour of a program text, but also because they do not satisfy criterion c3: there is a lot of reasoning prescribed for each (implicit or explicit) call of an escape procedure, see again appendix B.

Thus it seems worthwhile to reconsider the design of the escape mechanism, without weakening its power too much. Clearly the inattractive part of rule S2 is its third premise, necessitated by the appearance of escproc assumptions in the semantics of an escape procedure. It reflects that at the site of call, and not known from their definitions, escape procedures may turn out to be mutually activating, even if no simultaneous (recursive) definitions have been allowed!

Once the escproc assumptions have been eliminated from the escape procedure semantics assem, the third premise of S2 is no longer needed and we may merge the remainder of it together with S1 into the rules for procedure and escape procedure call:

\[ \text{let Env contain psem, esc e_i(val:ty_i) and escproc } E_i e_i{(y_i)} \]
and assume that y_i will not occur free in I and a, then
\[ \text{Env } \vdash \forall y_i:ty_i. (Q:h:th.E_i[f+a]\wedge I) \rightarrow E_i (i=1..n) \]
\[ \text{PC3 Env } \vdash (Q:h:th.P[f+a]\wedge I) p(a) \ (Q:h:th.Q[f+a]\wedge I) \]
where no actual var parameter occurs free in I

(*) equivalently: Env[y_i\rightarrow y'_i] + y_i:ty_i \vdash E_i \rightarrow E_i (i=1..n) where y'_i not free in Env, y_i.
and for escape procedure call a simple axiom suffices,

EP2: \( \text{Env} + \text{esc } e \ (\text{val}\ y\ y) + \text{escproc } E e[\{y\}] \vdash \{E[y\Rightarrow a]\} e(a)\{\text{false}\}. \)

It only remains to eliminate the \text{escproc} assumptions from esem. These may be eliminated either by undesirable "syntax restrictions" (like: escapes may not be used globally in escape procedures) or by the following change in the semantics, affecting rule E1 for escape procedure definition: escape procedure bodies, unlike procedure bodies, should be evaluated with a binding for their global escapes fixed in the environment \text{Env} in which the definition is considered. This leads to the following rule.

\[
\text{Let esem} \equiv \text{escproc } E e[y\Rightarrow y'] \vdash \text{without the } e, \text{asem } _1.
\]

\[
\begin{align*}
\text{Env} & \vdash [\text{escproc } e] + \text{esem}] [y\Rightarrow y'\Rightarrow y''] + \text{var } y\ y'\ y'' \vdash \{E[y,y',y'',y]\} \text{body} [S[y,y',y'',y]] \\
\text{Env} & \vdash [\text{escproc } e] + \text{esem} \vdash \{R\} \text{ stmt } \{S\}
\end{align*}
\]

E2

\[
\text{Env} \vdash \{R\} \text{ def } \text{escproc } e = \text{val } y \text{ in } \text{body} \text{ corp} \text{ in } \text{stmt } \text{ ni } \{S\}
\]

where \( y'' \) not free in \text{Env}, \text{y}.

The binding might be called "static relative to" the dynamic instance of the procedure body to which the definition is local, because \text{Env} may contain \text{escproc} semantics assumed, but not statically known, in the surrounding procedure body. Appendix D summarizes the rules. Appendix C contains a realistic example of the use of escapes, proved correct according to the new rules. (The program is also correct according to the original rules).

3.3. Conclusion

Based on the axiomatization of the escape mechanism it was felt that at least some features were too complicated. The attempt to simplify the semantics has been directed by a consideration of the formal rules. Very strikingly, it was only afterwards that we recognized that the formal simplification has resulted in an intuitive improvement as well: now the activation of an escape procedure really escapes the block to which it is local. Thus the above development of the escape mechanism is quite a nice demonstration of the influence of Hoare-like semantics on the design of programming language concepts, as advocated by Gries (1976) and others.

It might be interesting to note that the complexity in the original mechanism has resulted from an attempt to keep things simple: an exceptional kind of procedure, escape procedure, was our starting point and in order to have as few exceptions as possible escape procedure bodies were treated as much as possible like procedure bodies! This decision has now been changed. For the same reason the definition of an escape procedure was originally termed a declaration and we not only expected but also aimed at a similarity in the formalization of procedures and escape procedures. The axiomatizations however clearly show that we had better distinguish between the two concepts and their syntactic appearances.

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4. REFERENCES

Appendix A. A proof fully exploiting P1 and P1C

For each textpart the environment consists of the semantic descriptions connected to some vertical line in front of the textpart; it is just the usual notion of scope which has been visualized.
Appendix B. A proof essentially involving S2.

The environment has been indicated in the same way as in appendix A.

```
 esc one ( ), esc two ( ), var w

 def escproc {true} one { (escproc {w ≥ 0} two {()}) } 
 = if w < 3 then w := 1 {w = 1} 
    else w := w - 3; {w ≥ 0} two {} {false} {w = 1} 
    fi {w = 1 - implies the outer block's postcondition w = 1 \lor w = 2} 

 in def escproc {true} two { (escproc {w ≥ 3} one {()}) } 
 = if w < 2 then w := 2 {w = 2} 
    else w := w + 1; {w ≥ 3} one {} {false} {w = 2} 
    fi {w = 2 - the inner block's postcondition} 

 ni {true} two {} {false} {w = 2} 

 ni {w = 2} {w = 1 \lor w = 2}
```

Proof of calls on line (a) and (b) immediate by S1! We prove line (c) below. Because in each application of S2 (n = 1 and n_1 = 1 and) \( \tilde{I}_1 \) = true, we omit the second premise of S2.

To be proven Env \( \vdash \{true\} \ two \ {} \ {false} \), from (1) by EC1.

Note that escproc \( \{true\} \ two \ \{ (escproc \{w ≥ 0\} \ one \ {()} \} \} \in Env, \)

(1) Env \( \vdash \ \{\ \} \), from (2,3) by S2.

Note that escproc \( \{true\} \ one \ \{ (escproc \{w ≥ 3\} \ two \ {()} \} \} \in Env, \)

(2) Env \( \vdash \ w ≥ 0 \ \triangleright \ true \), assumed,

(3) Env + escproc \( \wedge w ≥ 0 \ one \ {()} \ \downarrow \ \{ (escproc \{w ≥ 3\} \ two \ {()} \} \), from (4,5) by S2.

Let Env + escproc \( \wedge w ≥ 0 \ one \ {()} \) be denoted by Env',

note that escproc \( \{true\} \ two \ \{ (escproc \{w ≥ 0\} \ one \ {()} \} \} \in Env', \)

(4) Env' \( \vdash \ w ≥ 3 \ \triangleright \ true \), assumed,

(5) Env' + escproc \( \wedge w ≥ 0 \ two \ {()} \ \downarrow \ \{ (escproc \{w ≥ 0\} \ one \ {()} \) \), by axiom S1.

**End of proof.**
Appendix C. Realistic use of escapes, proved correct according to section 3.3.

The environment has not explicitly been indicated.
Assume a (standard) environment containing:

```plaintext
esc overflow( ),
```

```plaintext
proc recip (var x val y {hyp y_0 escproc {y_0 too small} overflow {{ }}})
    {entry: y = y_0 , exit: x = 1/y_0}.
```

A program dealing with matrix inversion may read:

```plaintext
new esc sing( );
```

```plaintext
proc invert (var X val M {hyp M_0 escproc {M_0 singular} sing {{ }}})
    {entry: M = M_0 , exit: X = M_0^{-1}}
```

```plaintext
= def escproc {M_0 singular} overflow {{ }}
```

```plaintext
    = {M_0 singular} sing ( ) {false}{X = M_0^{-1} corp
```

```plaintext
in (* standard inverting, using recip for reciprocal: *)
```

```plaintext
........
(a too small ⇒ M_0 singular) ∧ .......}
```

```plaintext
∃y_0. (a = y_0) ∧ ((a = y_0) ∧ (y_0 too small ⇒ M_0 singular) ∧ ...)}
```

```plaintext
recip (x, a {∃y_0. (y_0 too small) ∧ (... ∧ (y_0 too small ⇒ M_0 sing)...})
```

```plaintext
  implies M_0 singular)
```

```plaintext
∃y_0. (x = 1/y_0) ∧ ((a = y_0) ∧ .........)}
```

```plaintext
{r = 1/a ∧ .......}
```

```plaintext
........
```

```plaintext
ni {X = M_0^{-1}}
```

```plaintext
corp
```

```plaintext
in : ;
```

```plaintext
(* inner part of a loop in which matrices are computed and inverses printed: *)
```

```plaintext
def escproc {i-th matrix singular} sing {{ }}
```

```plaintext
    = {i-th matrix singular} write (i, "singular") {blocks postcond} corp
```

```plaintext
in (* overflow caused by compute aborts at least this block! *)
```

```plaintext
compute (A);
```

```plaintext
{∃M_0. (A = M_0) ∧ (M_0 is i-th matrix)}
```

```plaintext
invert (A, A {M_0 singular ∧ M_0 is i-th matrix ⇒ i-th matrix singular});
```

```plaintext
{∃M_0. (A = M_0^{-1}) ∧ (M_0 is i-th matrix)}
```

```plaintext
{A is i-th inverse}
```

```plaintext
write (A)
```

```plaintext
ni {i-th inverse or singularity message printed}
```

```plaintext
:
```

```plaintext
:
```

```plaintext
ni.
```
Appendix D. Summary of the rules for the escapes.

Consider the following abbreviations:

\[ \text{psem} \equiv \text{proc} \{ P \} \ p(\text{spec} \ f'; t) \ (\text{hyp} \ h; \text{th} \ e_1 \text{assem}(i=1..n)) \ \{ Q \} \]

\[ e_1 \text{assem} \equiv \text{escproc} \{ E_1 \} \ e_1 \{(y_1)\} \]

where \( f', h, e_1 \) distinct and the formal val parameters not free in \( Q \) and the variables used globally by \( p \) free in \( \text{psem} \),

\[ \text{esem} \equiv \text{escproc} \{ E \} \ e \{(y')\} \]

\[ \text{initesem} \equiv \text{escproc} \{ \text{true} \} \ e \{(y')\}. \]

Escape variable declaration:

\[ \text{Env}[e'e'] + \text{esc} e \ (\text{val}ty) + \text{initesem} \vdash \{ \text{R[e'e']} \} \text{stmnt} \{ S[e'e'] \} \]

\[ \text{Env} \vdash \{ R \} \text{new} \text{esc} e \ (\text{val}ty) \\text{in} \text{stmnt} \ ni \{ S \} \]

where \( e' \) not free in \( \text{Env}, e \).

Procedure declaration:

\[ \text{Env}[p+p']-\{ \text{escproc} \} + \text{psem}[f+f''] + \text{var} f; t; h; \text{th} + e_1 \text{assem}(i=1..n) \]

\[ \vdash \{ p[f, f'+f'', f] \} \text{body} \{ Q[f, f'+f'', f] \} \]

\[ \text{Env}[p+p'] + \text{psem} \vdash \{ \text{R}[p+p'] \} \text{stmnt} \{ S[p+p'] \} \]

\[ \text{Env} \vdash \{ R \} \text{new} \text{proc} p \ (\text{spec} t; f) = \text{spec} f; t; f \text{ in body corp in stmnt} \ ni \{ S \} \]

where \( p' \) not free in \( \text{Env}, p \)

and \( h, f'' \) distinct and not free in \( \text{Env}, p, p', f. \)

Escape procedure definition:

\[ \text{Env} - \{ \text{escproc} e \} + \text{esem}[y+y''] + \text{var} y; ty \vdash \{ E[y, y'+y'', y] \} \text{body} \{ S[y, y'+y'', y] \} \]

\[ \text{Env} - \{ \text{escproc} e \} + \text{esem} \vdash \{ R \} \text{stmnt} \{ S \} \]

\[ \text{Env} \vdash \{ R \} \text{def} \text{escproc} e = \text{val} y \text{ in body corp in stmnt} \ ni \{ S \} \]

where \( y'' \) not free in \( \text{Env}, y. \)

Procedure call:

let \( \text{psem, esc} e_1(\text{val}ty_1), \text{escproc} \{ E_1 \} e_1(\{y_1\}) \in \text{Env} \)

and assume \( y_1 \) not free in \( I \) and \( a \), then

\[ \text{Env} \vdash y_1; ty_1. \ \{ \exists \text{h}; \text{th}. E_1[f+a] \wedge I \} \Rightarrow E_1(i=1..n) \]

\[ \text{Env} \vdash \{ \exists \text{h}; \text{th}. P[f+a] \wedge I \} \ p(a) \ { \exists \text{h}; \text{th}. Q[f+a] \wedge I } \]

provided no actual var parameter free in \( I \).

Escape procedure call:

\[ \text{Env} + \text{esc} e(\text{val}ty) + \text{escproc} \{ E \} e \{(y)\} \vdash \{ E[y+a] \} \ e(a) \{ \text{false} \}. \]