Associativity of Cartesian Product in Entity-Relation Diagrams

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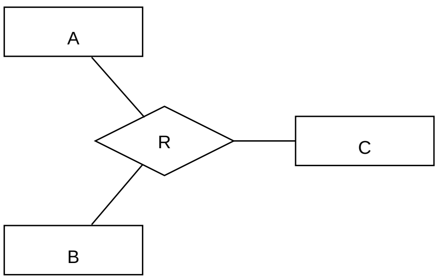
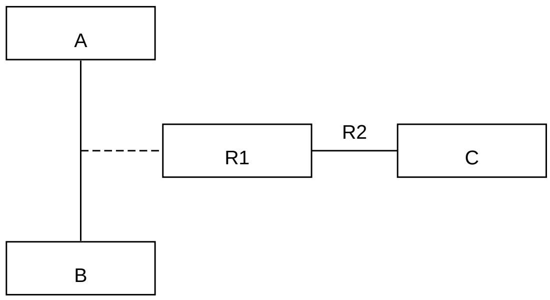
Associativity of Cartesian product is well-known in set theory:

$$A \times B \times C \quad \simeq \quad A \times (B \times C)$$

The formulation of this folklore fact in terms of Entity-Relationship diagrams is less known (but equally true). Below we present the formulation and proof. We use the UML notation for ERDs (class diagrams) with the convention that the absence of a multiplicity means 'no restriction', that is, '$0\mathinner{\ldotp\ldotp}*$' is the default multiplicity

Theorem. The following ERDs are equivalent:





The proof of the equivalence consists of showing that there exist transformations f and g, mapping the instantiations of the one ERD into those of the other ERD, in such a way that $f \circ g = id$ and $g \circ f = id$. Here are the definitions, using symbols R, R_1, R_2 to denote an arbitrary relation of the type R, R_1, R_2 of the ERDs above:

$$\begin{array}{lcl} f(R) & = & (\{a, b, c \mid (a, b, c) \in R \bullet (a, b)\}, \quad \{a, b, c \mid (a, b, c) \in R \bullet ((a, b), c)\}) \\ g(R_1, R_2) & = & \{a, b, c \mid (a, b) \in R_1 \wedge ((a, b), c) \in R_2 \bullet (a, b, c)\} \end{array}$$

It is easy to show that $f \circ g = id$ and $g \circ f = id$.

Remark. The left-hand side of the theorem is symmetric in A, B, C, whereas the right-hand side is not. Consequently, we can interchange letters A, B, C in order to get two more variants of the theorem.